

Neutral pseudoscalar meson decays: $\pi^0 \rightarrow \gamma\gamma$ and $\eta \rightarrow \gamma\gamma$ in $SU(3)$ limit**

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Abstract

Present and planned experiments motivate new theoretical study of properties of light unflavoured pseudoscalar meson decays. An overview including details on two-loop calculation in $SU(3)$ limit is given.

Keywords: chiral perturbation theory, radiative decay of π^0 , higher-order correction

1. Introduction

We would like to study unflavoured decays of light neutral pseudoscalar mesons. This reduces the particle content to π^0 , η and eventually η' , ruling out K^0 decays that violate hypercharge conservation and are suppressed by G_F^2 (two-photon decays are further suppressed by α^2 compared to hadronic ones). Standard model is thus reduced to QCD (extended eventually only by QED corrections) which is successfully described by an effective theory known as chiral perturbation theory (ChPT).

The π^0 meson being the lightest meson cannot decay to other hadronic states. Its dominant decay mode (with more than 98% probability) is $\pi^0 \rightarrow \gamma\gamma$ and is connected with the Adler-Bell-Jackiw triangle anomaly [1]. The $\pi^0\gamma\gamma$ vertex is closely connected with other allowed π^0 decay modes: $e^+e^-\gamma$, $e^+e^-e^+e^-$, e^+e^- (with branching ratios [2]: $0.01174(35)$, $3.34(16)\times 10^{-5}$, $6.46(33)\times 10^{-8}$, respectively). In order to describe these processes with sufficient precision one can employ two-flavour ChPT at appropriate order. This can simply incorporate corrections to the current algebra result attributed either to $m_{u,d}$ masses or electromagnetic corrections with other effects hidden in the low energy constants (LECs). Naively, two-flavour ChPT should converge very fast and next-to-leading order (NLO) should be sufficient from the

point of view of today's experiments. However, as we are exploring the anomalous sector which is poorly known, phenomenologically richer $SU(3)$ ChPT must be also used in order to obtain numerical prediction for low energy constants. This on the other hand enables to describe $\eta \rightarrow \gamma\gamma$ in the same framework.

The motivation for our study is both theoretical and experimental. As mentioned, $\pi^0 \rightarrow \gamma\gamma$ represents (probably) the most important example of the triangle anomaly in quantum field theory. It is interesting that at NLO the amplitude gets no chiral corrections from the so-called chiral logarithms [3] and this motivate the calculation at NNLO even for $SU(2)$ ChPT as was done in [4]. It was found that there are indeed chiral logarithms generated by two-loop diagrams, but they are relatively small. It turns one's attention back to NLO order and contributions proportional to LECs. To this end the phenomenology of $\eta \rightarrow \gamma\gamma$ and inevitably $\eta - \eta'$ mixing must be employed. We intend to do the full two-loop calculation of both $\pi^0 \rightarrow \gamma\gamma$ and $\eta \rightarrow \gamma\gamma$ in three-flavour ChPT. As a first step we will present here the calculation and result in the $SU(3)$ limit, i.e. for $m_u = m_d = m_s$.

From the experimental side let us mention the PrimEx experiment at JLab. It is designed to perform the most precise measurement of the neutral pion lifetime using the Primakoff effect (for first run results see [5]). After JLab's 12 GeV upgrade the extension of the experiment for η and η' radiative width measurements is planned. Neutral pion decay modes were studied with interesting results at KTeV and it is promising to measure them in forthcoming NA62 at CERN.

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2. Chiral expansion

Let us briefly summarize main points of ChPT, for details see [6]. Starting point is the chiral symmetry of QCD, called chiral because it acts differently on left and right-handed quarks, which is exact for $m_{u,d,s} = 0$:

$$G = SU(3)_L \times SU(3)_R,$$

where we dropped $U(1)_A$ which is not a good symmetry due the anomaly. However, this anomaly is proportional to a divergence which must thus vanish in any order of perturbation theory. We are touching the problem referred as $U(1)$ problem and we will avoid further discussion assuming that the ninth axial current is really not conserved and a possible divergence term is not present in QCD Lagrangian (referred itself as strong CP problem). Assuming further confinement it can be proven that the axial subgroup of G is spontaneously broken and the associated 8 Goldstone bosons can be identified with pions, kaons and eta. The real non-zero masses of u, d, s quarks, explicit symmetry breaking, are added as a perturbation and this expansion around the chiral limit together with the momentum expansion is referred to as ChPT. Standard power counting assumes that $m_{u,d,s} = O(p^2)$, and Lorentz invariance implies that only even powers of derivatives (p) can occur. The leading order (LO) thus starts at $O(p^2)$ and one can have only tree diagrams. The next-to-leading order (NLO) is $O(p^4)$ and can include one-loop contribution and similarly next-to-next-to-leading order (NNLO) is $O(p^6)$ and can have up to two-loop diagrams. The last important point to be discussed here is the so-called chiral or external anomaly which would correctly incorporate the full symmetry pattern of QCD. It is connected with the fact that quarks carry also electromagnetic charge. In fact some Green functions of QCD (e.g. VVA) are not invariant under chiral symmetry, the difference was calculated first by Bardeen [1] and incorporated to the action by Wess, Zumino and Witten (WZW) [7]. This action starts at $O(p^4)$ and thus the anomalous vertex shifts our counting by one order (i.e. NNLO here is $O(p^8)$).

3. Decay modes

We are primarily interested now in two-photon decays of π^0 and η . Nevertheless let us summarize shortly their “spin-off” products, namely

- $\pi^0 \rightarrow e^+e^-\gamma$ so called Dalitz decay is important in normalization of rare pion and kaon decays. This was supported by its precise and stable prediction: for 30 years its official PDG value was same (based

on LAMPF experiment). However the last edition changed this number, based on ALEPH results and so it will have impact in other measurements via the normalization. The differential decay rate is discussed in [8].

- $\pi^0 \rightarrow e^+e^-e^+e^-$ or double Dalitz decay enables experimental verification of π^0 parity. KTeV set recently new limits on parity and CPT violation [9]
- $\pi^0 \rightarrow e^+e^-$ depends directly only on fully off-shell $\pi^0\gamma^*\gamma^*$ vertex. KTeV measurement [10] is off by 3.5σ from the existing models. It can set valuable limits on models beyond SM
- $\pi^0 \rightarrow \text{invisible}(\gamma)$, exotics and violation processes were also studied in π^0 decays. It includes mainly decay to neutrinos but is also interesting in beyond SM scenarios (neutralinos, extra-light neutral vector particle, etc.)

(for more references cf. [2]). The same modes are also possible in η decays, see e.g. [11].

4. LO and NLO calculation

In the chiral limit the decay width is fixed by axial anomaly with the result

$$\Gamma(\pi^0 \rightarrow \gamma\gamma)^{CA} = \frac{m_{\pi^0}^3}{64\pi} \left(\frac{\alpha N_C}{3\pi F_\pi} \right)^2 \approx 7.76 \text{ eV}. \quad (1)$$

It is in excellent agreement with experiment, which is the opposite situation to two-photon η decay. In $SU(3)$ limit (and also in chiral limit) the two studied amplitudes are connected by Wigner-Eckart theorem $\sqrt{3}T_\eta = T_{\pi^0}$, i.e.

$$\Gamma(\eta \rightarrow \gamma\gamma)^{CA} = \frac{m_\eta^3}{64\pi} \left(\frac{\alpha N_C}{3\sqrt{3}\pi F_\pi} \right)^2 \approx 173 \text{ eV}, \quad (2)$$

which is far from experiment $0.510 \pm 0.026 \text{ keV}$ [2]. (Note that using F_η instead of F_π makes this difference even larger.) The difference is attributed to η - η' mixing. At NLO order, apart from tree diagrams coming from WZW and $O(p^6)$ odd-parity Lagrangian, we should include two one-loop topologies (depicted in Fig.1).

The full one-loop calculation based on wave function renormalization and chiral expansion of masses and de-

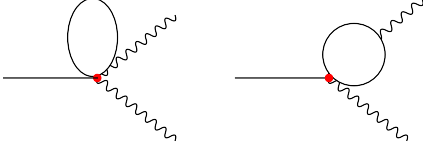


Figure 1: One-loop corrections to two photon pseudoscalar decays. A (red) dot represents the odd-parity coupling.

cay constants leads to:

$$\begin{aligned}\Gamma(\pi^0 \rightarrow \gamma\gamma)^{NLO} &= \Gamma(\pi^0 \rightarrow \gamma\gamma)^{CA} \times \left[1 - \frac{256\pi^2}{3} m_\pi^2 C_7^{Wr} \right]^2, \\ \Gamma(\eta \rightarrow \gamma\gamma)^{NLO} &= \Gamma(\eta \rightarrow \gamma\gamma)^{CA} \times \left[\frac{F_\pi^2}{F_\eta^2} + \frac{256\pi^2}{9} \right. \\ &\quad \times \left. \left(4m_K^2 - 7m_\pi^2 \right) C_7^{Wr} + 24(m_K^2 - m_\pi^2) C_8^{Wr} \right]^2.\end{aligned}\quad (3)$$

Note, as anticipated, the very simple, polynomial form of the results without logarithms. This is especially accomplished by correct replacement of F_0 , i.e. $F_0 \rightarrow F_\pi$ and F_η in π^0 and η decay respectively.

It is clear from (3) that η - η' mixing must be hidden in C_8^W LEC. A rough estimate using resonance saturation suggests that C_8^W must be much bigger than C_7^W . For further discussion see [12] and [4].

5. Two-loop calculation in $SU(3)$ limit

The $O(p^8)$, (or equivalently NNLO, or two-loop) calculation was already performed for $\pi^0 \rightarrow \gamma\gamma$ in two-flavour ChPT. Natural extension for $SU(3)$ will supply us with both π^0 and also $\eta \rightarrow \gamma\gamma$ and enable to test and verify chiral expansion in odd intrinsic sector (cf. study for even sector [13]). It is, however, clear that this calculation will be difficult: we are facing instead of one, three different scales in overlapping two-loop diagrams (sunset and vertex). Big effort was already given in the simpler two-point (sunset) case, and we still lack general analytic form. We plan to calculate it using method described in [14] but we need to go beyond the loop integrals computed there. There exists, however, apart from chiral limit, one non-trivial limit which can be used to obtain analytical result as it depends again only on one scale. It is an $SU(3)$ limit, where we set $m_u = m_d = m_s = m \neq 0$. This we can simply connect with $O(p^2)$ mass: $M_\pi^{(0)2} = 2Bm$.

The current algebra prediction, fixed by the anomaly, is free from any mass contribution. The mass enters explicitly at NLO order only, and therefore to obtain NNLO order we need to connect $O(p^2)$ parameter with

physical (referring to a world where $m_u = m_d = m_s$) $SU(3)$ mass:

$$\frac{M_\pi^2}{F_\pi^2} = 1 + \frac{M_\pi^2}{F_\pi^2} \left[\frac{L}{3} - 8(3L_4^r + L_5^r - 6L_6^r - 2L_8^r) \right] + O(M_\pi^4)$$

with chiral logarithm defined as $(4\pi)^2 L = \ln M_\pi^2/\mu^2$. On the other hand connection of F_0 with physical $SU(3)$ decay constants is needed up to NNLO order

$$\frac{F_\pi}{F_0} = 1 + \frac{M_\pi^2}{F_\pi^2} (12L_4^r + 4L_5^r - \frac{3}{2}L) + \frac{M_\pi^4}{F_\pi^4} f_{NNLO} + O(M_\pi^6).$$

The NNLO part was already calculated in general $SU(N_F)$ in [15] and for our $N_F = 3$ is given by

$$f_{NNLO} = \frac{\lambda_F}{(4\pi)^2} + \bar{\lambda}_F + \mathcal{K}_F + r_F + \frac{1561L}{288(4\pi)^2} - \frac{421}{2304(4\pi)^4}$$

with

$$\begin{aligned}\lambda_F &= -2L_1^r - 9L_2^r - 7/3L_3^r \\ \bar{\lambda}_F &= 8(3L_4^r + L_5^r)(21L_4^r + 7L_5^r - 24L_6^r - 8L_8^r) \\ \mathcal{K}_F &= 1/2(34\mathcal{K}_1 + 13\mathcal{K}_2 + 13\mathcal{K}_3 - 45\mathcal{K}_4 - 15\mathcal{K}_5) \\ r_F &= 8(C_{14}^r + 3C_{15}^r + 3C_{16}^r + C_{17}^r)\end{aligned}$$

and $\mathcal{K}_i = (4L_i^r - \Gamma_i L)L$ using renormalization coefficients taken from [6].

As already mentioned, for $SU(3)$ limit π^0 and η decays are related by Wigner-Eckart theorem and we thus need to calculate only one of these processes. Following Weinberg power-counting at NNLO we need to consider *a)* tree graphs with either *a)* one vertex from odd $O(p^8)$ sector or *a)* one from odd (even) $O(p^6)$ and second from even (odd) $O(p^4)$; *b)* one-loop diagrams with one vertex with NLO coupling (even or odd) and *c)* the two-loop graphs with one vertex taken from the WZW Lagrangian. All other vertices should be generated by the $O(p^2)$ chiral Lagrangian.

Case *a)* is treated via wave function renormalization. However, the odd-sector Lagrangian at $O(p^8)$ for three flavours has not yet been studied. The connected LEC will be denoted as D_i^W and set only a posteriori to cancel all local divergences. Concerning one-loop Feynman diagrams, we have already summarized them in Fig.1, for NNLO the topology stays the same, we need just to insert higher-order vertices. Non-trivial part of calculation is hidden in two loops. The Feynman diagrams to deal with are summarized in Fig.2. Corrections (tadpoles) to propagators are not depicted. Note that the most of diagrams are the same as in the two-flavour case. As anticipated by the nature of the anomaly there is one new topology (the last one diagram in Fig.2) with

anomalous vertex without direct photon insertion (so-called Chesire-cat smile). Of course, into these graphs one should insert all possible combinations of pions, kaons and eta (fortunately in $SU(3)$ limit with identical masses).

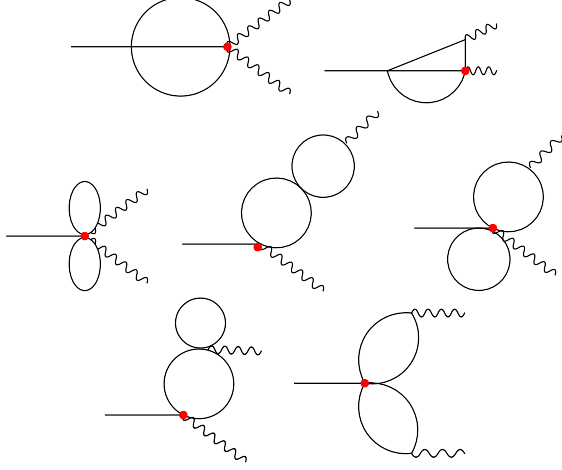


Figure 2: Two-loop corrections to two photon pseudoscalar decays.

We summarize the preliminary result in the following form (T is normalized as $T^{CA} = 1$ at LO, cf. eqs (3)).

$$\frac{F_\pi^4}{m_\pi^4} T^{NNLO} = \frac{\lambda}{(4\pi)^2} + (4\pi)^2 \bar{\lambda} + (4\pi)^2 \mathcal{K} + r + \frac{329L}{96(4\pi)^2} + \frac{9\sqrt{3}Cl_2(\pi/3) - 4\zeta(3) - \frac{7093}{1152}}{(4\pi)^4} \quad (4)$$

with

$$\lambda = 0$$

$$\bar{\lambda} = -\frac{256}{3} F_0^2 C_7^{Wr} (3L_4^r + L_5^r - 3L_6^r - L_8^r)$$

$$\mathcal{K} = 4\mathcal{K}_4^w + 10\mathcal{K}_7^w - 2\mathcal{K}_9^w + 4\mathcal{K}_{11}^w - \frac{1}{2}\mathcal{K}_{13}^w - 2\mathcal{K}_{14}^w - \mathcal{K}_{15}^w$$

$$r = -32C_{12}^r - 96C_{13}^r - 4D_{lim}^{Wr}$$

and $\mathcal{K}_i^w = (4F_0^2 C_i^{Wr} - \eta_i^{(3)} L) L$ using renormalization coefficients taken from [16]. The $O(p^8)$ chiral coupling which would cancel local divergences in $SU(3)$ limit is denoted by D_{lim}^W and our exact calculation fixes its decomposition

$$D_{lim}^W = \frac{(c\mu)^{2(d-4)}}{F_0^2} \left[D_{lim}^{Wr}(\mu) + \Lambda^2 \frac{127}{12} + \Lambda \left(\frac{208}{3} L_1^r + 32L_2^r + \frac{248}{9} L_3^r + 36L_4^r + 12L_5^r + \frac{91}{128(4\pi)^2} + (4\pi F_0)^2 (8C_4^{Wr} + \frac{100}{9} C_7^{Wr} - 4C_9^{Wr} + 8C_{11}^{Wr} - C_{13}^{Wr} - 4C_{14}^{Wr} - 2C_{15}^{Wr}) \right) \right]$$

6. Conclusion

We have summarized here our preliminary results concerning a two-loop calculation of $\pi^0(\eta) \rightarrow \gamma\gamma$ in $SU(3)$ limit (where $m_{u,d} = m_s = m$). The word preliminary refers also to the fact that independent calculation with physical masses is in progress [17] and it should allow us to crosscheck here presented result in this limit. The possibility of studying two-photon decays of light-meson on lattice was very recently demonstrated in [18]. The simple analytical result can be very useful in this direction as one can vary masses without changing LECs.

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